# Final Review, Part 1

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December 5, 2017

## 1 What we observe

- **Radio galaxies.** These are galaxies that are "loud" in the radio due to synchrotron processes. Quasars and blazars are related objects that may also be very luminous in the radio band.
- **Molecules.** Molecule rotation modes are a classic example of what can be observed in radio frequencies. We use radio telescopes to study protoplanetary disk and cloud chemistry by looking for molecules like CO. Note that we cannot see any molecule that does not have a permanent dipole moment.
- **Solar system objects.** As you learned in a lab activity, the Sun can (theoretically) be observed in radio wavelengths. Jupiter and its moons have also been observed this way.
- **Cosmic microwave background.** The CMB is an almost-perfect blackbody source that emits in the radio, as you observed in another lab activity.
- **Masers.** As we learned just recently, masers can emit in the radio. A maser is an astrophysical phenomenon wherein molecules are more abundant in an "upper" state than in a "lower" state and therefore emit, rather than absorb, radiation.
- **Can you think of others?**

## 2 Important equations

### 2.1 Rayleigh-Jeans limit

Brightness temperature $T_B$ specifies the flux $F_\nu$ uniquely at a given frequency $\nu$ by

$$F_\nu = \frac{2\nu^2}{c^2} k_B T_B. \tag{1}$$

*Remember that brightness temperature is not a physical temperature except in the case of a perfect blackbody.* Rather, It is the temperature that a blackbody would need to have at a given frequency to produce the observed flux. Also remember that the above formula only applies in the limit of long wavelengths, such as what we observe in radio.

## 2.2 Nyquist theorem

The power observed from a source of temperature $T$ is

$$P = k_B T \Delta\nu \tag{2}$$

where $k_B$ is the Boltzmann constant and $\Delta\nu$ is the bandwidth. Remember to multiply by gain $G$ if necessary!

## 2.3 Nyquist sampling theorem

The minimum time between samples that we need to completely recover the information in a signal is given by

$$\Delta\tau = \frac{1}{2\Delta\nu}. \tag{3}$$

## 2.4 Ruze law

The aperture "efficiency" $\eta$ is given by the product of all the efficiencies of relevant effects in the system. For example,

$$\eta = \eta_{\text{taper}}\eta_{\text{spillover}}\eta_{\text{surface}}\eta_{\text{blockage}}\eta_{\text{loss}}\cdots \tag{4}$$

gives just some of the effects that may be at play. The Ruze formula is

$$\eta_{\text{surface}} = \frac{\langle A_0 \rangle}{A_{\text{geom}}} = e^{-\left(\frac{4\pi\sigma_\epsilon}{\lambda}\right)^2}. \tag{5}$$

A typical specification is $\sigma_\epsilon/\lambda = 20$. Here, $\sigma_\epsilon$ is the rms deviation of the aperture surface from a parabola.

## 2.5 Radiometer equation

The radiometer equation predicts how the temperature of a system fluctuates about the mean. It is given by

$$\Delta T = \frac{T_{\text{rx}} + T_A}{\sqrt{\tau\Delta\nu}}, \tag{6}$$

where $T_{\text{rx}}$ is the receiver noise temperature, $T_A$ is the antenna temperature, $\tau$ is the integration time, and $\Delta\nu$ is the bandwidth. It is important to remember, however, that there is an "implicit" dependence on the system gain $G$ in this equation. $\Delta T$ has a linear dependence on $G$, so higher gain will amplify both the receiver and antenna temperatures!

## 2.6 Friis equation

In a signal path that includes multiple components, the Friis equation predicts the overall noise temperature of the system due to the combined effects of all components.

$$T_{\text{noise}} = T_{\text{rx1}} + T_{\text{rx2}}/G_1 + T_{\text{rx3}}/G_1 G_2 + \cdots \tag{7}$$

where $G_n$ is the gain of the $n$th component and $T_{\text{rx}n}$ is its noise temperature. The practical implications of the Friis equation is that we should put low-noise components closest to the signal input, because higher-noise components will have their noise contributions scaled by the inverse of the gain of the components before them. We demonstrated this in the radiometer lab.

## 2.7 Fourier transform

The Fourier transform converts a function from "real" space to "frequency" space; in radio, the real variable is time and its conjugate is frequency. For some signal $f(t)$,

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, \mathrm{d}t. \tag{8}$$

We can also denote the Fourier transform of $f$ as $\mathcal{F}[f]$. In this notation, the following properties hold:

$$\mathcal{F}[af] = a\mathcal{F}[f] \;\; \text{(scaling)} \tag{9}$$

$$\mathcal{F}[f + g] = \mathcal{F}[f] + \mathcal{F}[g] \;\; \text{(addition)} \tag{10}$$

$$\mathcal{F}[f(t - a)] = e^{-2\pi i a\omega}\mathcal{F}[f(t)] \;\; \text{(shift)} \tag{11}$$

$$\mathcal{F}\left[\frac{\partial f}{\partial t}\right] = 2\pi i\omega\mathcal{F}[f] \;\; \text{(derivative)} \tag{12}$$

$$\mathcal{F}[f * g] = \mathcal{F}[f]\,\mathcal{F}[g] \;\; \text{(convolution)} \tag{13}$$

$$\mathcal{F}[f \star g] = \mathcal{F}[f]^*\,\mathcal{F}[g] \;\; \text{(cross-correlation)} \tag{14}$$

Review the Fourier transforms of some common functions, like the Gaussian, Dirac delta, box, and Dirac comb (or shah) functions.

## 2.8 Plancherel's theorem

This theorem is a statement that the total "power" in a function is equal to the total "power" in its Fourier transform.

$$\int_{-\infty}^{\infty} |f(t)|^2 \,\mathrm{d}t = \int_{-\infty}^{\infty} \left|\hat{f}(\omega)\right|^2 \mathrm{d}\omega \tag{15}$$

## 2.9 Hankel transform

The Hankel transform is the axisymmetric equivalent of a two-dimensional Fourier transform. In the limit that the angular extent of the source is small ($\sin\theta \approx \theta$), the transform is given by

$$\tilde{W}(\theta) = 2\pi \int W(q)\, J_0(2\pi q\theta)\, q\,\mathrm{d}q \tag{16}$$

and its inverse is

$$W(q) = 2\pi \int \tilde{W}(\theta)\, J_0(2\pi q\theta)\, \theta\,\mathrm{d}\theta. \tag{17}$$